

Problem Set

- There are 4 questions (see also the second page)
- Submit your answers **typed** (not handwritten)
- To write your answers use free tools for latex like overleaf.com or use suitable software on your computer (it is all free); knowledge of latex is required for this doctoral position.

Question # 1

Solve this limit

$$\lim_{x \rightarrow a} \frac{x^x - x^a}{a^x - a^a}. \quad (1)$$

Show all your calculations and also plot the limit as x approaches a .

Question # 2

Define an orthonormal basis of vectors in \mathbb{R}^n .

- How many vectors are in the basis? Explain why.
- Separate the basis vectors into two sets \mathbb{A} and \mathbb{B} . Can the vectors in \mathbb{A} be linearly dependent to any vector in \mathbb{B} ? Explain why.
- Define the square matrix $P = [A \ \mathbf{0}]$, where A is the matrix of basis vectors from the set \mathbb{A} in the previous question and $\mathbf{0}$ is a matrix of zeros. Compute $P^T P$. Show all your calculations.

Question # 3

Write the explicit formula of the gradient of

$$E[u] = \sum_{i=1}^1 \sum_{j=0}^1 \cos(u[i, j]) \sin(u[i-1, j]) \quad (2)$$

with respect to the variable u , which is a 2×2 matrix with indices from 0 to 1 in both coordinates. Show all the steps of your calculations.

Question # 4

Consider IID samples x_1, \dots, x_m that are Poisson distributed with mean λ .

- Write the probability mass function $p(x)$ of the Poisson distribution and show all the steps to compute the mean λ explicitly as the expectation of x .
- Use the given samples to find the maximum likelihood estimate for the parameter λ . Show all the steps of your calculations.