Problem Set

- There are 4 questions (see also the second page)
- Submit your answers typed (not handwritten)
- To write your answers use free tools for latex like overleaf.com or use suitable software on your computer (it is all free); knowledge of latex is required for this doctoral position.

Question #1

Solve this limit

$$\lim_{x \to a} \frac{x^x - x^a}{a^x - a^a}.$$
(1)

Show all your calculations and also plot the limit as x approaches a.

Question #2

Define an orthonormal basis of vectors in \mathbb{R}^n .

- How many vectors are in the basis? Explain why.
- Separate the basis vectors into two sets A and B. Can the vectors in A be linearly dependent to any vector in B? Explain why.
- Define the square matrix P = [A 0], where A if the matrix of basis vectors from the set A in the previous question and 0 is a matrix of zeros. Compute P^TP. Show all your calculations.

Question #3

Write the explicit formula of the gradient of

$$E[u] = \sum_{i=1}^{1} \sum_{j=0}^{1} \cos(u[i,j]) \sin(u[i-1,j])$$
(2)

with respect to the variable u, which is a 2×2 matrix with indices from 0 to 1 in both coordinates. Show all the steps of your calculations.

Question #4

Consider IID samples x_1, \ldots, x_m that are Poisson distributed with mean λ .

- Write the probability density function p(x) of the Poisson distribution and show all the steps to compute the mean λ explicitly as the expectation of x
- Use the given samples to find the maximum likelihood estimate for the parameter λ . Show all the steps of your calculations.