Questionnaire

Question # 1

Consider points $x \in \mathbb{R}^{19}$ satisfying the following equation

$$w^\top x + b = 0 \tag{1}$$

for some parameters w and b.

Questions:

1) What is the minimum possible dimensionality of w and of b, if the equation describes a hyperplane?

2) Assume that w is a 19×2000 random matrix, where each entry is a random sample from the Normal distribution, and $b = 0$. What is a likely solution in x of the above equation?

3) Assume that the above equation describes a hyperplane. Prove that w is normal to the hyperplane.

Question # 2

Write the explicit formula of the gradient of

$$E[u] = \sum_{i=2}^{n-1} \sum_{j=2}^{m-1} \sin\left[(u[i+1,j] - u[i,j-1])^2 + \epsilon \right] \tag{2}$$

with respect to the variable u, which is an $n \times m$ matrix, and where $\epsilon > 0$ is a given constant. Show all the steps of your calculations.

Question # 3

Write the explicit formula of the maximum likelihood estimator for the parameter α of the following probability density distribution

$$p(x; \alpha, \epsilon) = \frac{1}{Z} e^{-\alpha|x-\epsilon|}, \tag{3}$$

where Z is the partition function, and $\alpha > 0$ and ϵ are unknown scalars. Assume that you are given m independent and identically distributed samples $x^{(1)}, \ldots, x^{(m)}$. Show all the steps of your calculations.