Problem Set

Problem # 1

Assume that $x_0$ is a solution to the following linear system

$$Ax_0 = b$$

where $A$ is an $n \times m$ matrix, with $n < m$, $x_0$ is an $m$-dimensional column vector and $b$ is an $n$-dimensional column vector. Show that the $L^2$ norm of $x_0$ can be arbitrarily larger than the norms of $A$ and $b$.

Problem # 2

Write the explicit formula of the gradient of

$$E[u] = \sum_{i=2}^{n-1} \sum_{j=2}^{m-1} \cos \left( (u[i+1,j] - u[i,j-1])^2 \right)$$

with respect to the variable $u$, which is an $n \times m$ matrix. Show all the steps of your calculations.

Problem # 3

Write the explicit formula of the maximum likelihood estimator for the covariance matrix $\Sigma \in \mathbb{R}^{n \times n}$ of the following probability density function

$$p(x; \mu, \Sigma) \propto e^{-\frac{(x - \mu)\top \Sigma^{-1}(x - \mu)}{2}}$$

given $m$ independent and identically distributed samples $x^{(1)}, \ldots, x^{(m)}$. Assume that the covariance $\Sigma$ is diagonal, but the non-zero values can be all different from each other. Show all the steps of your calculations and justify them. Do not just write the final formula.